InterWrite

information architecture
content strategy
online design

Houghton Mifflin Company
Math Interactive Lessons

Tutorial 34: Transformations of Graphs and Functions

02/13/04

Author: Shawn Hackshaw

Tutorial: Transformations of Graphs of Functions *(34)*

Outline

p01	Prepare for the Tutorial (includes PrepTests)
m01x01	Explore the Concept
m01	Concept: Horizontal Translations (or Shifts)
m01c01	Study the Concept
m01e01	Try an Example
m01p01	Practice Exercises
m02	Concept: Vertical Translations (or Shifts)
m02c01	Study the concept
m02e01	Try an Example
m02p01	Practice Exercises
m03	Concept: Reflections
m03c01	Study the Concept
m03e01	Try an Example
m03p01	Practice Exercises
m03v01	Video clip
mas01	Mastery Tests

Prepare for the Tutorial

Objectives

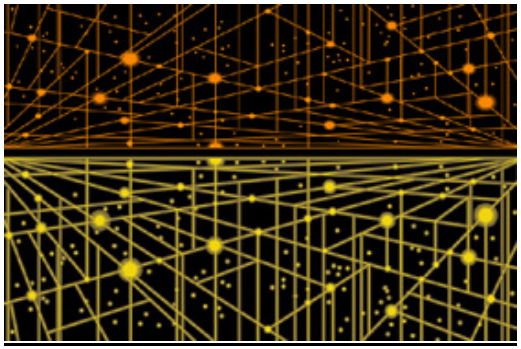
In this tutorial, you will learn the following concepts:

- Vertical Translations of Graphs
- Horizontal Translations of Graphs
- Reflections of Graphs

Prep Tests

You should be comfortable with the following concepts *before* you take this Tutorial. Click to take a Prep Test.

- Graphing Functions
- Graphing a Line Given Its Point and Slope



Computer graphics artists have specialized software to help them create three-dimensional drawings. In this image, the artist used the software to reflect and shift images to create patterns.

View Image Credit

ID: p01 Photo: il0034p01ph01.jpg

Tutorial: Transformations of Graphs of Functions p01

Prepare for the Tutorial: Prep Tests

Graphing Functions

il0032c01pre01.xml
Beginning Algebra with Applications, 6/E [HMBA]
Aufmann; 0618306129

1. 5.5.2.56 Graphs of Linear Functions (MC)
2. 5.5.2.59 Graphs of Linear Functions (MC)

Graphing a Line Given a Point and the Slope

il0026c01pre01.xml
Elementary Algebra: Discovery and Visualization, 3/E [HMHE]
Hubbard/Robinson; 0618223932

1. 4.4.2.142 Sketching a Graph Given It's Slope and a Point (MC)
2. 4.4.2.143 Sketching a Graph Given It's Slope and a Point (MC)

Concept: Vertical Translations (or Shifts) m01x01

Explore the Concept

ID= il0034m01x01.swf

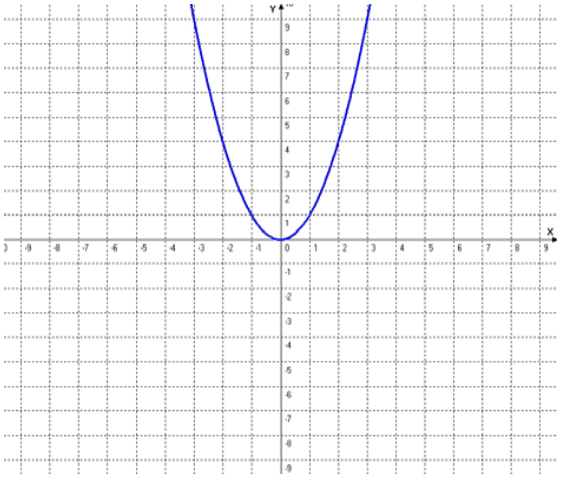
TRANSLATIONS OF GRAPHS

The standard form of the equation for a parabola is $y = (x - h)^2 + k$. The vertex is at (h,k).

Using your mouse, click on the arrow buttons to move the graph around on the coordinate grid. You can also click on the vertex of the parabola and drag it around the coordinate plane.

Notice how the equation of the parabola changes depending on the placement of the vertex.

$$y = (x - h)^2 + k$$



[These arrows appear to the right of the coordinate grid]

Programming

1. Screen open with the graph of $y = x^2$.
2. Also on screen are radio buttons with arrows for up, down, left and right.
3. In the upper right-hand corner of the screen is the function $y = x^2$
4. The user interacts with the graph by means of the arrow buttons.
5. If the user presses the up button, the function description changes to $y = x^2 +$ the number of clicks of the button. (e.g. user presses the up arrow 3 times, the function description changes to $y = x^2 + 3$) Also, with each click of a button, the graph will move in that direction so that the user can see what happens to the graph as well as how it changes the equation for the graph.
6. If user presses the down button, the same as #5 above, except the value of the number of clicks is subtracted from the function description. (e.g. the graph is already at $y = x^2 + 3$, the user clicks the down arrow 5 times…as the graph moves down, the function description changes from $y = x^2 + 3$, to $y = x^2 + 2$, to $y = x^2 + 1$, to $y = x^2$, to $y = x^2 - 1$, to $y = x^2 - 2$)
7. If user presses the right arrow button, the function description changes to $y = (x -$ number of right arrow clicks$)^2 \pm$ number of up/down arrow clicks (e.g. if the graph begins at $y = x^2$, and the user presses the right arrow button 2 times, the graph moves to the right 2 units and the function descriptor changes to $y = (x-2)^2$. If at this point the user clicks up 3 times, the descriptor changes to reflect that and the graph moves. The descriptor would now read $y = (x-2)^2 + 3$.

8. If user presses the left arrow button, the function description changes to

$y = (x +$ number of left arrow clicks$)^2 \pm$ number of up/down arrow clicks (e.g. if the graph begins at $y = x^2$, and the user presses the left arrow button 2 times, the graph moves to the left 2 units and the function descriptor changes to $y = (x + 2)^2$. If at this point the user clicks down 3 times, the descriptor changes to reflect that and the graph moves. The descriptor would now read $y = (x + 2)^2$ - 3.

ALSO, IF PROGRAMMING ISN'T TOO DIFFICULT, A DRAG AND DROP INTERACTION COULD ALSO BE INCLUDED THAT WOULD ALLOW STUDENTS TO PICK UP THE PARABOLA AND PLACE IT ANYWHERE ON THE GRAPH AND HAVE THE EQATION SHOW THE SAME CHANGES BUT IT WOULD BE A FASTER MOVEMENT THAN WITH THE BUTTONS AND STUDENTS COULD SEE HOW THE MOVEMENT AFFECTS THE EQUATION.

Concept: Horizontal Translations (or Shifts) m01c01

Study the Concept *(Storyboard)*

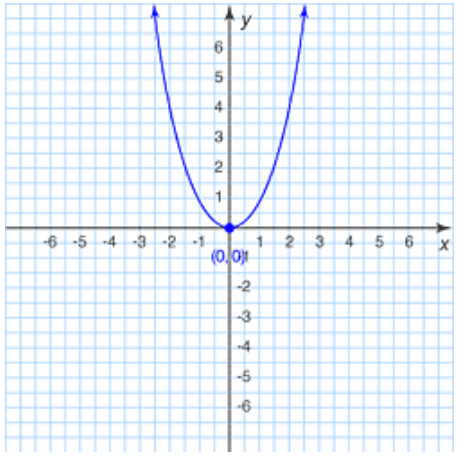
(*Animation Note: Each of these graphs are faded in sequentially accumulating the final combined image.)*

Horizontal Translations (or Shifts) of a Graph

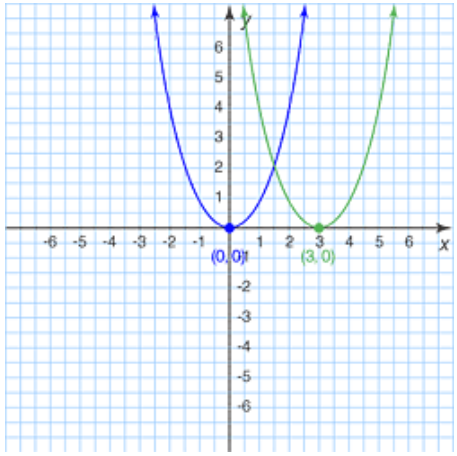
If f is a function and c is a positive constant, then

$y = f(x + c)$ is the graph of $y = f(x)$ shifted to the left c units.

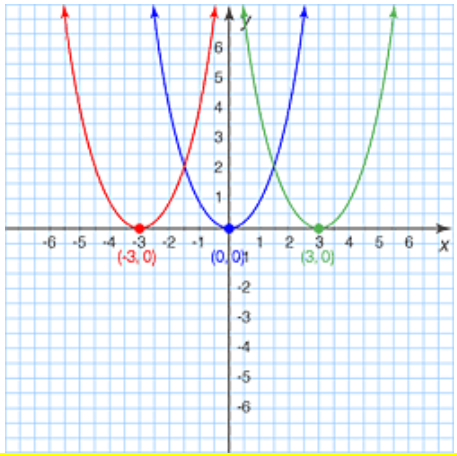
$y = f(x - c)$ is the graph of $y = f(x)$ shifted to the right c units.



The graph in **blue** represents $f(x) = x^2$.



The graph in **green** represents $f(x-3)=(x-3)^2$.



The graph in **red** represents $f(x+3)=(x+3)^2$.

Take Note

Take Note

One way to determine the direction of the horizontal shift is to ask, "For what value of *x* is $f(x)=0$." When $f(x)=(x-3)^2$, the value of *x* that would make the function equal to 0 is *x=3*. Therefore, the graph of

Take Note

$f(x-3)=(x-3)^2$ is the graph of $f(x)=x^2$ shifted to the right three units.

Pedagogical Tags: For the concept, example and practice for
Horizontal Translations (or Shifts) of Graphs
Graphing Calculator: Yes
Applications: None
ISBNs 0-618-39184-3, 0-618-15686-0, 0-618-10337-6, 0-618-38836-2, 0-618-38826-5, 0-618-38845-1

Concept: Horizontal Translations (or Shifts) m01c01

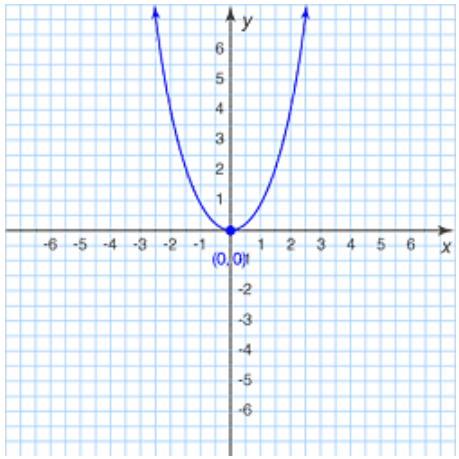
Study the Concept *(Programming and Text Script)*

File Names This Module (Study the Concept)
ID: m03c01
Audio: il0034m01c01.mp3

Animation: il0034m01c01anim.swf

Animation Frame 1: il0034m01c01animF1_IW.ai
Animation Frame 2: il0034m01c01animF2_IW. ai
Animation Frame 2: il0034m01c01animF3_IW. ai

Images, Text, Programming **Each numbered element is faded in unless otherwise noted**	**Narration (Text Transcript)**	**Narration (Audio Transcript)**	**Project Note**
1) (Title) Horizontal Translations (or Shifts) of a Graph	(No additional Text)	(No Audio)	
1a)(Definition Box) If *f* is a function and *c* is a positive constant, then	If *f* is a function and *c* is a positive constant, then	If *f* is a function and *c* is a positive constant, then y equals f of the sum x plus c is the graph of y equals f of x shifted to the left c units.	
1b)(Definition Box) $y = f(x + c)$ is the graph of $y = f(x)$ shifted to the left c units.	$y = f(x + c)$ is the graph of $y = f(x)$ shifted to the left c units.		
1c)(Definition Box) $y = f(x - c)$ is the graph of $y = f(x)$ shifted to the right c units.	$y = f(x - c)$ is the graph of $y = f(x)$ shifted to the right c units.	y equals f of the difference of x and c is the graph of y equals f of x shifted to the right c units.	
2) (Animation) (This part appears at the opening of the lesson)	(No Text)	(No Audio)	



3) (Main Screen Text)

The graph in **blue** represents $f(x) = x^2$.

The graph in **blue** represents $f(x) = x^2$.

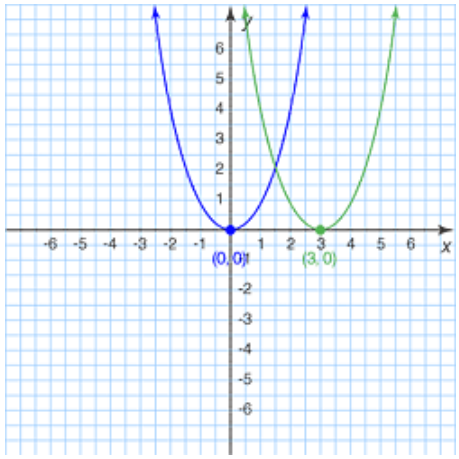
The graph in **blue** represents the function f of x equals the x squared.

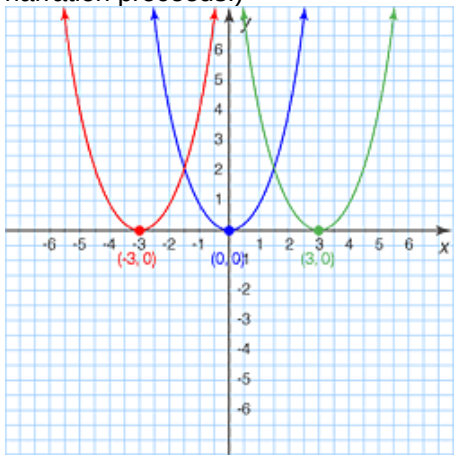
Notice that the vertex of the parabola is at the origin.

4) (Animation) (The purple graph appears as narration proceeds.)

(No additional Text)

(No Audio)

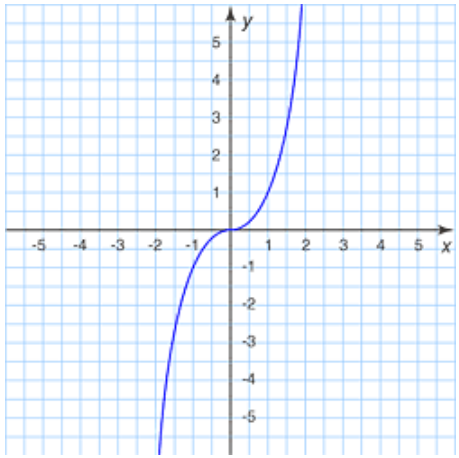


5) (Main Screen Text) The graph in **green** represents $f(x-3)=(x-3)^2$.	The graph in **green** represents $f(x-3)=(x-3)^2$.	The graph in **green** represents the function, f of the difference of x and three equals x minus three, quantity squared. Notice that the vertex has moved three units to the right.
6) (Animation) (The green graph appears as narration proceeds.)  	(No additional Text)	(No Audio)
7) (Main Screen Text) The graph in **red** represents $f(x+3)=(x+3)^2$.	The graph in **red** represents $f(x+3)=(x+3)^2$.	The graph in **red** represents the function, f of the sum x plus three, equals x plus three, quantity squared. Notice that the vertex has moved two units to the left.
8) (Take Note appears when the button is clicked) Take Note One way to determine the direction of the horizontal shift is to ask, "For what value of x is $f(x)=0$." When $f(x)=(x-3)^2$, the value	Take Note button	(No Audio for take note)

of x that would make the function equal to 0 is $x=3$. Therefore, the graph of $f(x-3)=(x-3)^2$ is the graph of $f(x)=x^2$ shifted to the right three units.

Concept: Horizontal Translations (or Shifts) m01e01

Try an Example *(Storyboard)*



Given the graph of $y = x^3$, describe the graph that represents $y = (x-5)^3$.

a. the graph shifts left five units

b. the graph shifts right five units

Check Answer (button)

Answer Feedback

The correct answer is b. The graph shifts to the right five units.

Incorrect Feedback

[if user chooses a]

Remember, a positive quantity added to the variable shifts the graph left and a positive quantity subtracted from the variable shifts the graph to the right. Please try again.

2nd wrong answer:

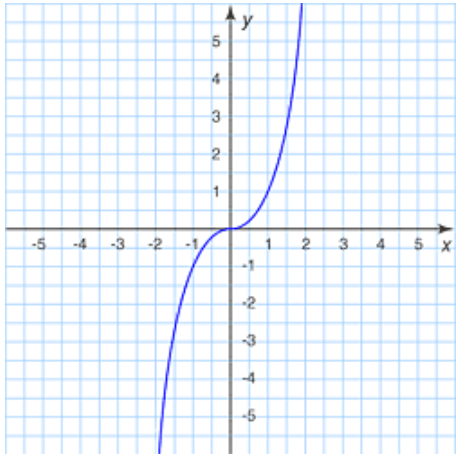
The correct answer is b. The graph shifts to the right five units.

Concept: Horizontal Translations (or Shifts) m01e01

Try an Example *(Programming and Text Script)*

File Names This Module (Try an Example)

ID: m03e01
Audio: il0034m01e01.mp3
Animation: il0034m01e01anim.swf
Animation Frame 1: il0034m01e01animF1_IW.ai

Images, Text, Programming Each numbered row is faded in unless otherwise noted	Narration (Text Transcript)	Narration (Audio Transcript)	Project Note
1) (Graphic) (If you are using a graphing utility, the equation for this graph is $y = x^3$) Given the graph of $y = x^3$, describe the graph that represents $y = (x-5)^3$.  	Given the graph of $y = x^3$, describe the graph that represents $y = (x-5)^3$.	Given the graph of y equals x cubed, describe the graph that represents y equal x minus five quantity cubed.	

2) (Main Screen Text) a. the graph shifts left five units b. the graph shifts right five units	a. the graph shifts left five units b. the graph shifts right five units	Choose from the following choices:
3) **) Check Answer** (button) **Answer Feedback** The correct answer is b. The graph shifts to the right five units.	**Check Answer** (button) **Answer Feedback** The correct answer is b. The graph shifts to the right five units.	The correct answer is b. The graph shifts to the right five units.
4)(Incorrect Feedback #1) [if user chooses a] Remember, a positive quantity added to the variable shifts the graph left and a positive quantity subtracted from the variable shifts the graph to the right. Please try again.	[if user chooses a] Remember, a positive quantity added to the variable shifts the graph left and a positive quantity subtracted from the variable shifts the graph to the right. Please try again.	[if user chooses a] Remember, a positive quantity added to the variable shifts the graph left and a positive quantity subtracted from the variable shifts the graph to the right. Please try again.
5)(Incorrect Feedback #2) The correct answer is b. The graph shifts to the right five units.	Sorry. You are incorrect. The correct answer is b. The graph shifts to the right five units.	The correct answer is b. The graph shifts to the right five units.

Concept: Horizontal Translations (or Shifts) m01p01

Practice Exercises

ID = m01p01

aufmann_ia/chap9/sect2/prob5

aufmann_ia/chay9/sect2/prob8

Concept: Vertical Translations (or Shifts) m02c01

Study the Concept *(Storyboard)*

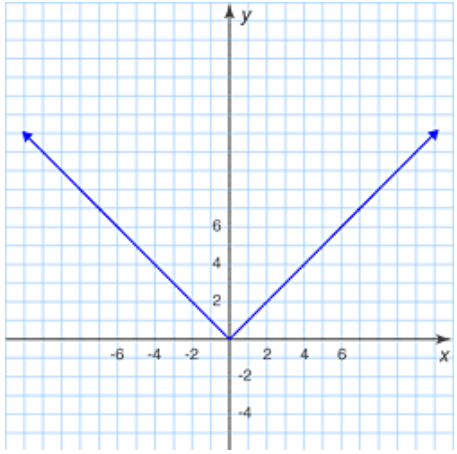
(Animation Note: These graphs lines fade in one at a time, but the final image has all 3 graphs.)

Vertical Translations (or Shifts) of a Graph

If f is a function and c is a positive constant, then

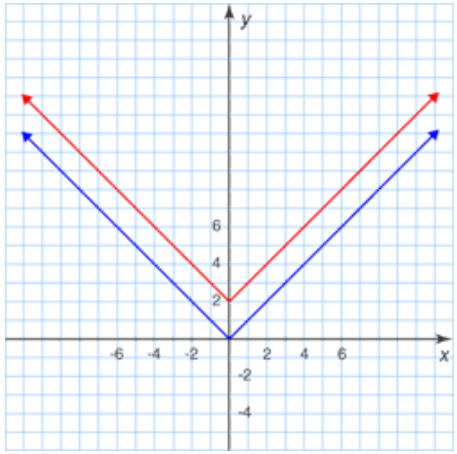
$y = f(x) + c$ is the graph of $y = f(x)$ shifted up c units.

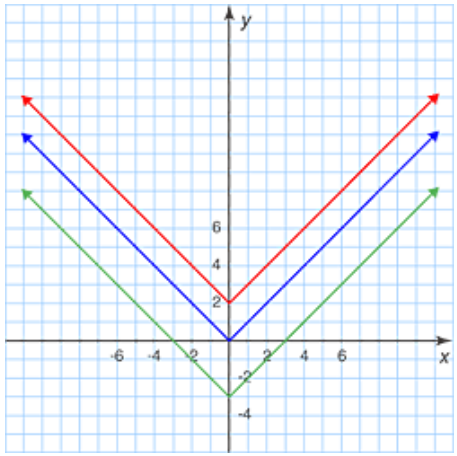
$y = f(x) - c$ is the graph of $y = f(x)$ shifted down c units.



The graph in **blue** represents $f(x) = |x|$.

The graph in **red** represents $f(x) + 2 = |x| + 2$.





The graph in **green** represents $f(x)-3=|x|-3$.

Pedagogical Tags: For the concept, example, explore the concept, and practice for Vertical Translations (or Shifts) of Graphs
Graphing Calculator: Yes
Applications: None
ISBNs 0-618-39184-3, 0-618-15686-0, 0-618-10337-6, 0-618-38836-2, 0-618-38826-5, 0-618-38845-1

Concept: Vertical Translations (or Shifts) m02c01

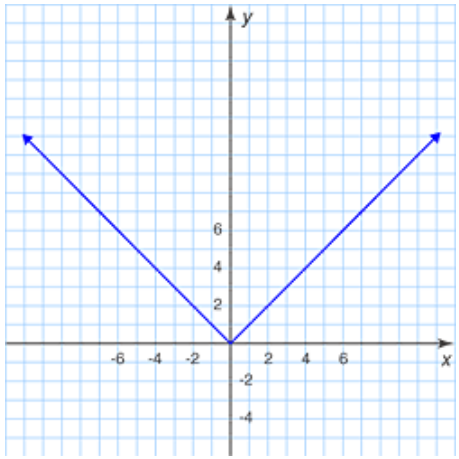
Study the Concept *(Programming and Text Script)*

File Names This Module (Study the Concept)
ID: m02c01
Audio: il0034m02c01.mp3

Animation: il0034m02c01anim.swf

Animation Frame 1: il0034m02c01animF1_IW.ai
Animation Frame 1: il0034m02c01animF2_IW.ai
Animation Frame 1: il0034m02c01animF3_IW.ai

Images, Text, Programming **Each numbered element is faded in unless otherwise noted**	**Narration (Text Transcript)**	**Narration (Audio Transcript)**	**Project Note**
1) (Title) Vertical Translations (or Shifts) of a Graph	(No additional Text)	(No Audio)	
1a)(Definition Box) If *f* is a function and *c* is a positive constant, then	If *f* is a function and *c* is a positive constant,	If *f* is a function and *c* is a positive constant,	
1b)(Definition Box) $y = f(x) + c$ is the graph of $y = f(x)$ shifted up c units.	then $y = f(x) + c$ is the graph of $y = f(x)$ shifted up c units.	then y equals f of x plus some quantity c is the graph of y equals f of x shifted up c units.	
1c)(Definition Box) $y = f(x) - c$ is the graph of $y = f(x)$ shifted down c units.	$y = f(x) - c$ is the graph of $y = f(x)$ shifted down c units.	y equals f of x minus some quantity c is the graph of y equals f of x shifted down c units.	
2) (Animation) (This part appears at the opening of the lesson)	(No Text)	(No Audio)	



3) (Main Screen Text)

The graph in **blue** represents $f(x) = |x|$.

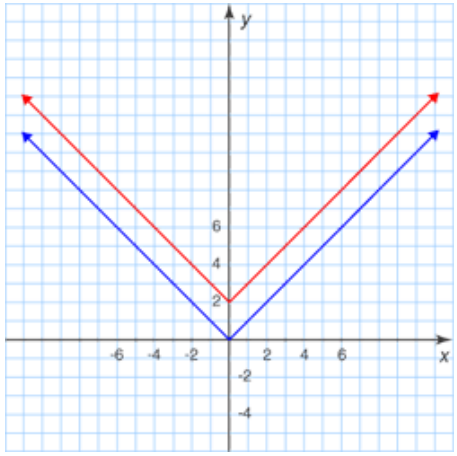
The graph in **blue** represents $f(x) = |x|$.

The graph in **blue** represents the function f of x equals the absolute value of x.

4) (Animation) (The purple graph appears as narration proceeds.)

(No additional Text)

(No Audio)



5) (Main Screen Text)

The graph in **red** represents

The graph in **red** represents $f(x) + 2 = |x| + 2$.

The graph in **red** represents the function, f of x, plus two equals the

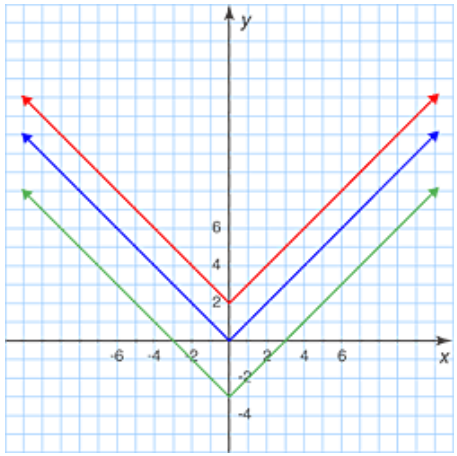
$f(x)+2=|x|+2$.

absolute value of x, plus two.

6) (Animation) (The green graph appears as narration proceeds.)

(No additional Text)

(No Audio)



7) (Main Screen Text)

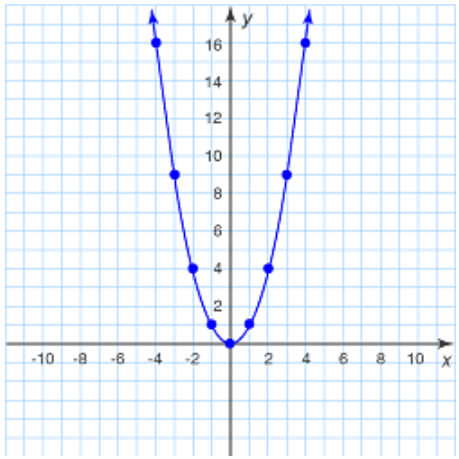
The graph in **green** represents $f(x)-3=|x|-3$.

The graph in **green** represents $f(x)-3=|x|-3$.

The graph in **green** represents the function, f of x, minus three, equals the absolute value of x, minus three.

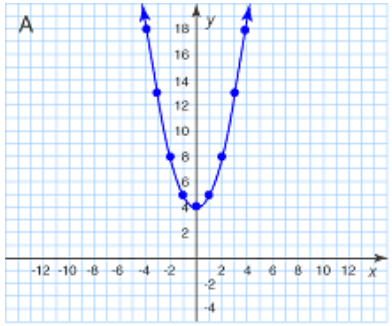
Concept: Vertical Translations (or Shifts) m02e01

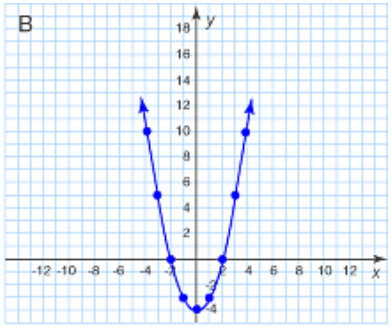
Try an Example 1 *(Storyboard)*



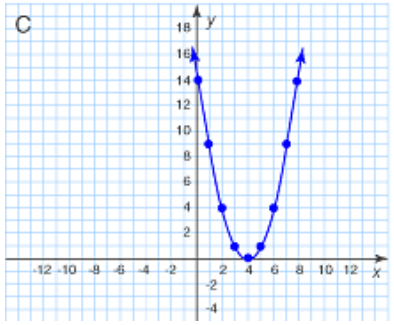
Given the graph of $f(x) = x^2$, select the graph that represents $g(x) = x^2 - 4$.

Click the graph below that represents the correct answer.

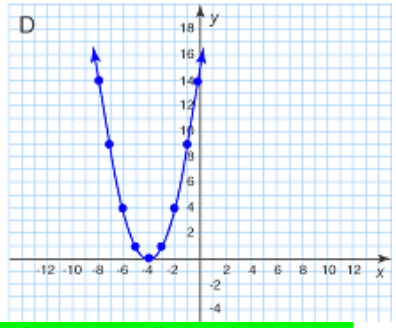




C



D



Check Answer (button)

Answer Feedback
Since the original graph has its vertex at the origin, subtracting 4 from every point on the graph brings the vertex to (0, -4).

Answer Feedback for Incorrect Answers.
[If user chooses A, C or D]

Remember, a positive quantity added to the function shifts the graph up and a positive quantity subtracted from the function shifts the graph down. Please try again.

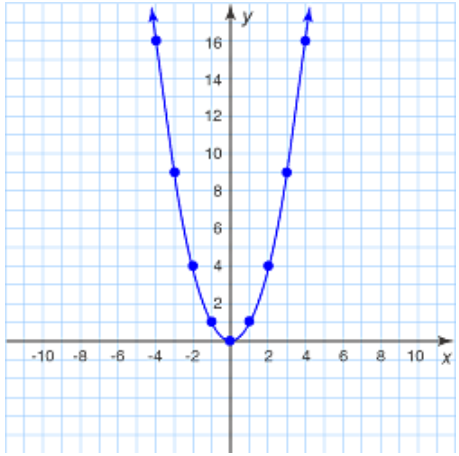
[incorrect feedback #2]
The correct answer is B. Since the original graph has its vertex at the origin, subtracting 4 from every point on the graph brings the vertex to (0, -4).

Concept: Vertical Translations (or Shifts) m02e01

Try an Example 1 *(Programming and Text Script)*

File Names This Module (Try an Example)
ID: m02e01
Audio: il0034m02e01.mp3

Animation: il0034m02e01anim.swf
Animation Frame 1: il0034m02e01animF1_IW.ai
Static frame 1 : il0034m02e01static01.swf
Static frame 2: il0034m02e01static02.swf
Static frame 3: il0034m02e01static03.swf
Static frame 4: il0034m02e01static04.swf

Images, Text, Programming **Each numbered element is faded in unless otherwise noted**	**Narration (Text Transcript)**	**Narration (Audio Transcript)**	**Project Note**
1)(Graphic) (If you are using a graphing utility, the equation for this graph is $y = x^2$) Given the graph of $f(x) = x^2$, select the graph that represents $g(x) = x^2 - 4$.  	Given the graph of $f(x) = x^2$, select the graph that represents $g(x) = x^2 - 4$.	Given the graph of f of x equals x squared, select the graph that represents g of x equals x squared minus four.	

2) (Main Screen Text)

Click the graph that represents the correct answer.

3) (Graphic or animation) (If we use an animation for this, fade in each graph individually, otherwise all four at once is fine.) (If you are using a graphing utility to make the graphs, the equations are as follows:

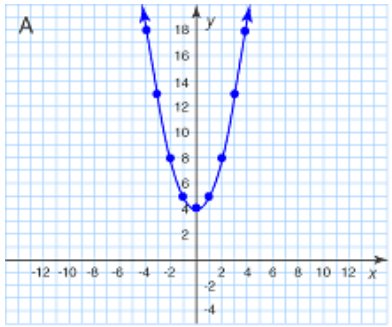
$A.\ y = x^2 + 4$

$B.\ y = x^2 - 4$

$C.\ y = (x-4)^2$

$D.\ y = (x+4)^2$

)



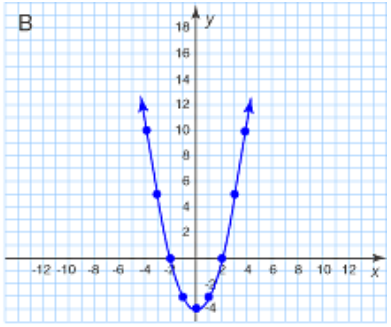
Click the graph below that represents the correct answer.

(No Text)

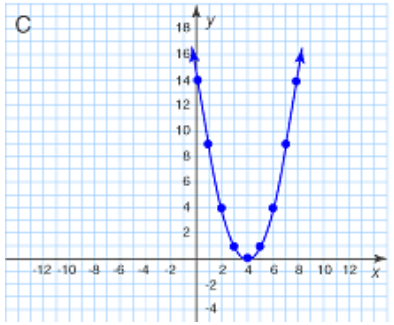
Click the graph below that represents the correct answer.

(No Audio)

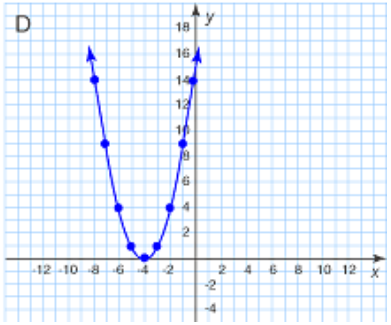
B



C



D



4) Check Answer (button)

Answer Feedback
The correct answer is B. Since the

Check Answer (button)

Answer Feedback
The correct answer is B. Since the

Since the original graph has its vertex at the origin, subtracting four from every point on the graph brings the vertex to the point zero, negative four.

original graph has its vertex at the origin, subtracting 4 from every point on the graph brings the vertex to (0, -4).	original graph has its vertex at the origin, subtracting 4 from every point on the graph brings the vertex to (0, -4).	
5) (Answer Feedback for Incorrect Answers) [If user chooses A, C or D] Remember, a positive quantity added to the function shifts the graph up and a positive quantity subtracted from the function shifts the graph down. Please try again.	[If user chooses A, C or D] Remember, a positive quantity added to the function shifts the graph up and a positive quantity subtracted from the function shifts the graph down. Please try again. .	[If user chooses A, C or D] Remember, a positive quantity added to the function shifts the graph up and a positive quantity subtracted from the function shifts the graph down. Please try again.
6) (Incorrect Feedback #2) The correct answer is B. Since the original graph has its vertex at the origin, subtracting 4 from every point on the graph brings the vertex to (0, -4).	The correct answer is B. Since the original graph has its vertex at the origin, subtracting 4 from every point on the graph brings the vertex to (0, -4).	The correct answer is B. Since the original graph has its vertex at the origin, subtracting four from every point on the graph brings the vertex to the point zero comma negative four.

Concept: Vertical Translations (or Shifts) m02p01

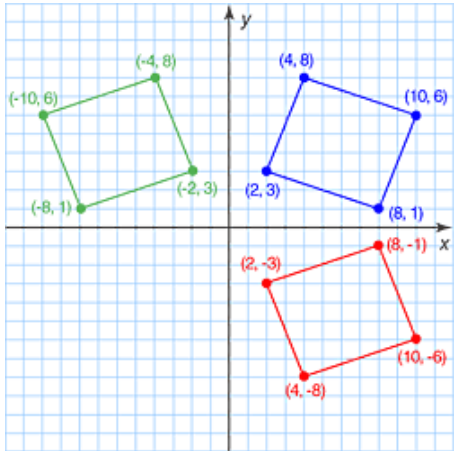
Practice Exercises

ID = m02p01

Aufmann_ia/chap9/sect2/prob4
Aufmann_ia/chap9/sect2/prob7
Larson_ca/chap2/sect5/prob5

Concept: Reflections m03c01

Study the Concept *(Storyboard)*



Reflections of a Graph

Reflections of the graph of $y = f(x)$ are represented as follows.

1. *Reflection about the x-axis*: $h(x) = -f(x)$

2. *Reflection about the y-axis*: $h(x) = f(-x)$

Take Note Button

You can picture a reflection as if you were holding a mirror up to the line of reflection. The points you must plot are what you see in the mirror.

Graphing a Reflection

To reflect a graph about the x-axis:

1. Choose several key points on the graph (such as vertices of a geometric shape.)

2. The points that are on the graph of the reflection will have the same *x*-coordinate and the *y*-coordinate will have the opposite sign as the original.

To reflect a graph about the y-axis:

1. Choose several key points on the graph.

2. The points that are on the graph of the reflection will have the same *y*-coordinate and the *x*-coordinate will have the opposite sign as the original.

Take Note Button

The red figure is a reflection about the *x*-axis of the blue figure.
The green figure is a reflection about the *y*-axis of the blue figure.

Pedagogical Tags: For the concept, example and practice for
Reflections
Graphing Calculator: No
Applications: None
ISBNs 0-618-39184-3, 0-618-15686-0, 0-618-10337-6, 0-618-38836-2, 0-618-38826-5, 0-618-38845-1

Concept: Reflections m03c01

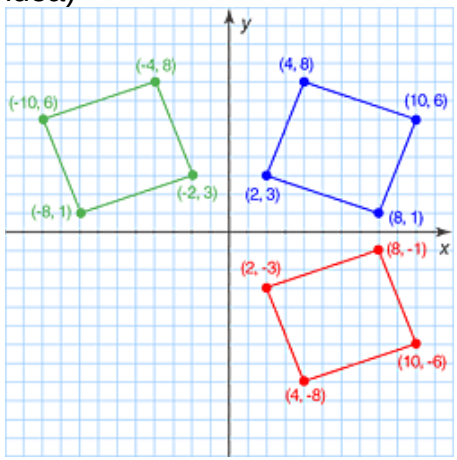
Study the Concept *(Programming and Text Script)*

File Names This Module (Study the Concept)

ID: m03c01
Audio: il0034m03c01.mp3
Animation: il0034m01c01anim.swf
Animation Frame 1: il0034m03c01animF1_IW.ai

Images, Text, Programming **Each numbered element is faded in unless otherwise noted**	**Narration (Text Transcript)**	**Project Note**
1) (Title) Graphing the Reflection of a Given Graph	(No Text)	
2) (Definition Box) **Reflections of a Graph**	(No Text)	
3) (Definition Box) Reflections of the graph of $y = f(x)$ are represented as follows.	Reflections of the graph of $y = f(x)$ are represented as follows.	
4) (Definition Box) **1. Reflection about the *x*-axis:** $h(x) = -f(x)$	The reflection of f of x about the x-axis is equal to the opposite of f of x.	
5) (Definition Box) **2. Reflection in the *y*-axis:** $h(x) = f(-x)$	The reflection of f of x about the y-axis is equal to f of negative x.	

6) (Animation) [when the narration mentions a reflection in the x axis, the purple figure should flash and when the narration talks about a reflection in the y axis, the green figure should flash…there are 2 lines for each then 1 line for each so users should get the idea)



(No Text)

7) (Procedure Box)

Graphing a Reflection

(No Text)

8) (Procedure Box)

[the **red** figure should flash until box 11]

To reflect a graph about the *x*-axis:

To reflect a graph about the x-axis

9) (Procedure Box)

1. Choose several key points on the graph (such as vertices of a geometric shape.)

First, you must choose key points on the original graph, for example, the vertices of a square.

10) (Procedure Box)

2. The points that are on the graph of the reflection will have the same *x*-coordinate, and the *y*-coordinate will have the opposite sign as the original.

Then, the points that are on the graph of the reflection will have the same *x*-coordinate, and the *y*-coordinate will have the opposite sign as the original.

11) (Procedure Box)

[the green figure should flash until end of box13]

To reflect a graph about the *y*-axis:

To reflect a graph about the y-axis

12) (Procedure Box)

1. Choose several key points on the graph.

First, you must choose key points on the original graph.

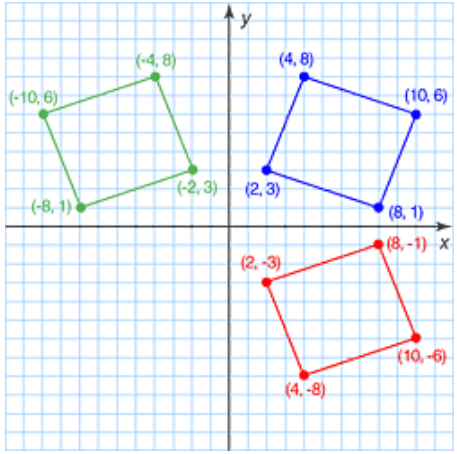
13) (Procedure Box) 2. The points that are on the graph of the reflection will have the same *y*-coordinate, and the *x*-coordinate will have the opposite sign as the original.	Then, the points that are on the graph of the reflection will have the same *y*-coordinate, and the *x*-coordinate will have the opposite sign as the original.
Take Note Button	
14) (Take Note Button appears when button is clicked) You can picture a reflection as if you were holding a mirror up to the line of reflection. The points you must plot are what you see in the mirror.	(No Text)
15) (Main Screen Text) [the purple figure should flash] The red figure is a reflection about the *x*-axis of the blue figure.	The red figure is a reflection about the *x*-axis of the blue figure.
16) (Main Screen Text) [the green figure should flash] The green figure is a reflection about the *y*-axis of the blue figure.	The green figure is a reflection about the *y*-axis of the blue figure.

Concept: Reflections m03c01

Study the Concept *(Programming and Audio Script)*

Images, Text, Programming	Narration (Audio Transcript)	Project Note
1) (Title) Graphing the Reflection of a Given Graph	(No Audio)	
2) (Definition Box)	(No Audio)	
Reflections of a Graph		
3) (Definition Box) Reflections of the graph of $y = f(x)$ are represented as follows.	Reflections of the graph of y equals f of x are represented as follows.	
4) (Definition Box) **1. Reflection about the *x*-axis:** $h(x) = -f(x)$	The reflection of f of x about the x-axis is equal to the opposite of f of x.	
5) (Definition Box) **2. Reflection in the *y*-axis:** $h(x) = f(-x)$	The reflection of f of x about the y-axis is equal to f of negative x.	
6) (Animation) [when the narration mentions a reflection in the x axis, the purple figure should flash and when the narration talks about a reflection in the y axis, the green figure should	(No Audio)	

flash…there are 2 lines for each then 1 line for each so users should get the idea)



7) (Procedure Box)

Graphing a Reflection

(No Audio)

8) (Procedure Box)

[the purple figure should flash until box 11]

To reflect a graph about the *x*-axis:

To reflect a graph about the x-axis

9) (Procedure Box)

1. Choose several key points on the graph (such as vertices of a geometric shape.)

First, you must choose key points on the original graph, for example, the vertices of a square.

10) (Procedure Box)

2. The points that are on the graph of the reflection will have the same *x*-coordinate, and the *y*-coordinate will have the opposite sign as the original.

Then, the points that are on the graph of the reflection will have the same *x*-coordinate, and the *y*-coordinate will have the opposite sign as the original.

11) (Procedure Box)

[the green figure should flash until end of box13]

To reflect a graph about the *y*-axis:

To reflect a graph about the y-axis

12) (Procedure Box)

1. Choose several key points on the graph.

First, you must choose key points on the original graph.

13) (Procedure Box)

2. The points that are on the graph of the reflection will have the same *y*-coordinate, and the *x*-coordinate will

Then, the points that are on the graph of the reflection will have the same *y*-coordinate, and the *x*-coordinate will have the opposite sign as the original.

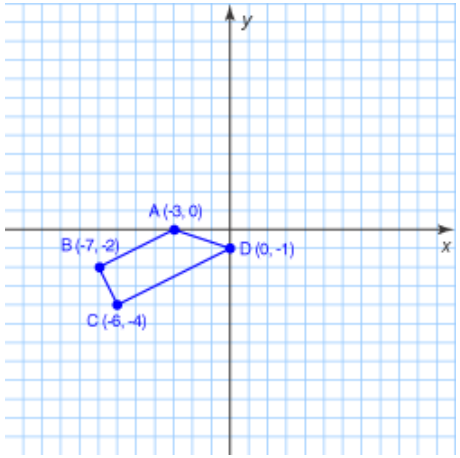
have the opposite sign as the original.

Take Note Button

14) (Take Note Button appears when button is clicked) You can picture a reflection as if you were holding a mirror up to the line of reflection. The points you must plot are what you see in the mirror.	(No Audio)
15) (Main Screen Text) [the purple figure should flash] The red figure is a reflection about the *x*-axis of the blue figure.	The red figure is a reflection about the *x*-axis of the blue figure.
16) (Main Screen Text) [the green figure should flash] The green figure is a reflection about the *y*-axis of the blue figure.	The green figure is a reflection about the *y*-axis of the blue figure.

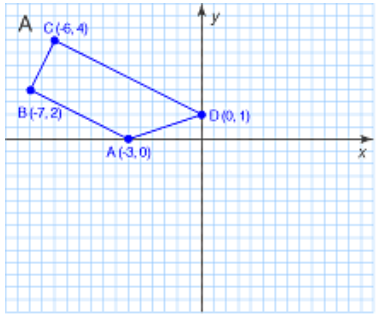
Concept: Reflections m03e01

Try an Example *(Storyboard)*

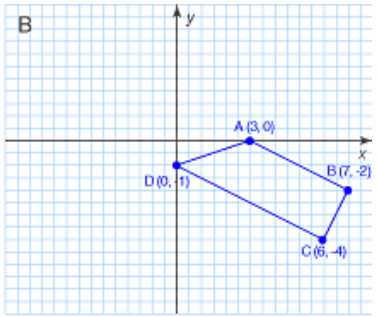


Using the graph of quadrilateral *ABCD*, select the graph that represents the reflection of the graph about the *y*-axis.

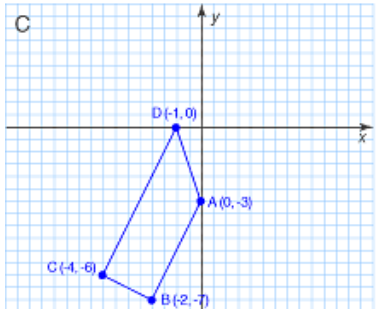
A



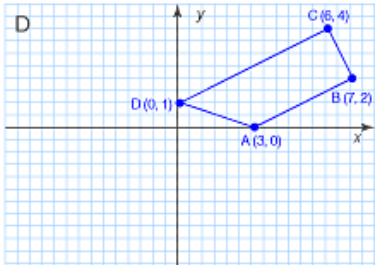
B



C



D



Check Answer (button)	
If user chooses B The *y* values have remained the same, and the *x* values have become the opposites of those in the original figure.	Feedback for incorrect answers. If A, C, or D is chosen: Remember that a reflection about the *y*-axis changes only the value of the *x*-coordinates. Please try again. Incorrect feedback #2 The correct answer is B. Notice that the *y* values have remained the same and the *x* values have become the opposites of those in the original figure.

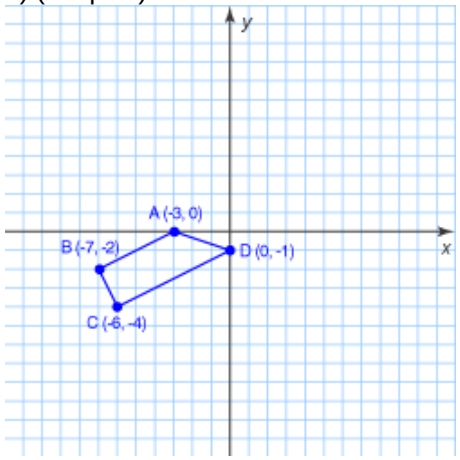
Concept: Reflections m03e01

Try an Example *(Programming and Text Script)*

File Names This Module (Study the Concept)

ID: m03e01
Audio: il0034m03e01.mp3

Animation: il0034m03e01anim.swf
Animation Frame 1: il0034m03e01animF1_IW.ai
Static Frame 1: il0034m03e01static01.swf
Static Frame 2: il0034m03e01static02.swf
Static Frame 3: il0034m03e01static03.swf
Static Frame 4: il0034m03e01static04.swf

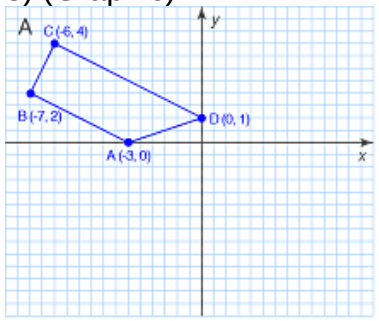
Images, Text, Programming **Each numbered element is faded in unless otherwise noted**	**Narration (Text Transcript)**	**Project Note**
1) (Graphic)  	(No Text)	
2) (Main Screen Text) Using the graph of quadrilateral ABCD, graph the reflection of the graph about	Using the graph of quadrilateral *ABCD*, select the graph that represents the reflection of the graph about the *y*-axis.	

the *y*-axis.

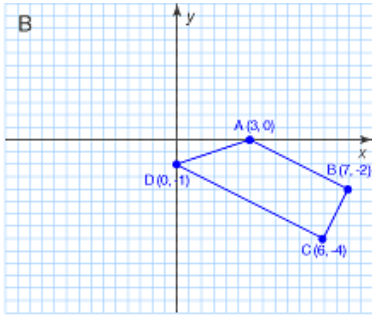
Click the graph that represents the correct answer.

3) (Graphic) (No Text)

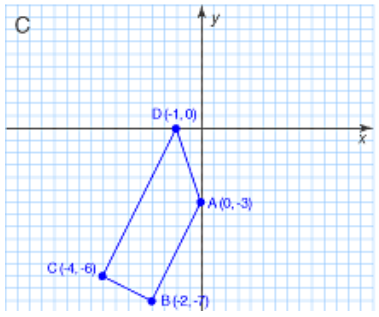
A



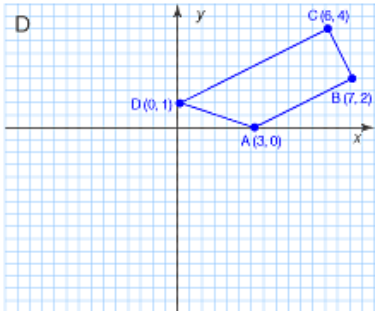
B



C



D



4) (*Answer feedback. Appears when button is pushed)*
Answer Feedback
The *y* values have remained the same, and the *x* values have become the opposites of those in the original figure.

The *y* values have remained the same, and the *x* values have become the opposites of those in the original figure.

5) (Answer feedback for incorrect answers)
[If A, C, or D is chosen:]

Sorry, you are not correct.
Remember that a reflection about the *y*-axis changes only the value of the *x*-coordinates. Please try again.

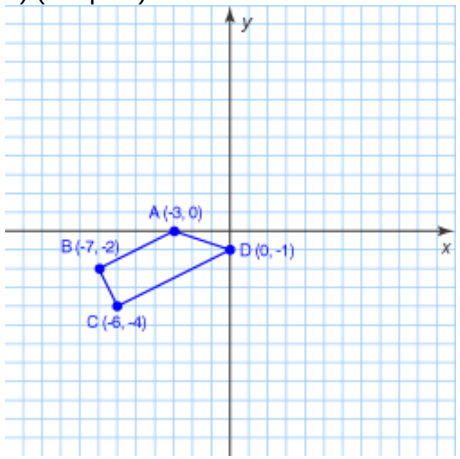
Remember that a reflection about the *y*-axis changes only the value of the *x*-coordinates. Please try again.

6) (incorrect feedback #2)
The correct answer is B. Notice that the *y* values have remained the same and the *x* values have become the opposites of those in the original figure.

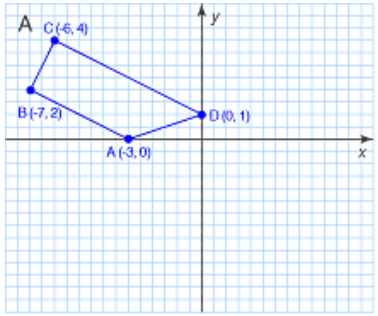
The correct answer is B. Notice that the *y* values have remained the same and the *x* values have become the opposites of those in the original figure.

Concept: Reflections m03e01

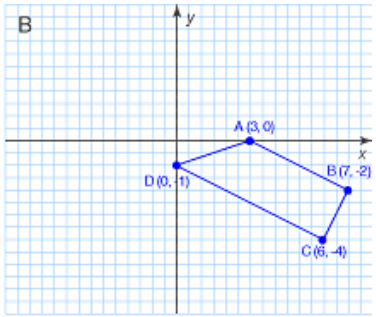
Try an Example *(Programming and Audio Script)*

Images, Text, Programming **Each numbered element is faded in unless otherwise noted**	**Narration (Audio Transcript)**	**Project Note**
1) (Graphic)  	(No Text)	
2) (Main Screen Text) Using the graph of quadrilateral ABCD, graph the reflection of the graph about the *y*-axis. Click the graph that represents the correct answer.	Using the graph of quadrilateral *ABCD*, select the graph that represents the reflection of the graph about the *y*-axis.	
3) (Graphic)	(No Text)	

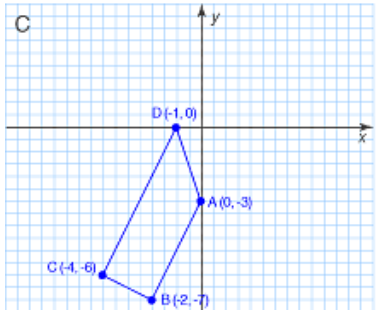
A



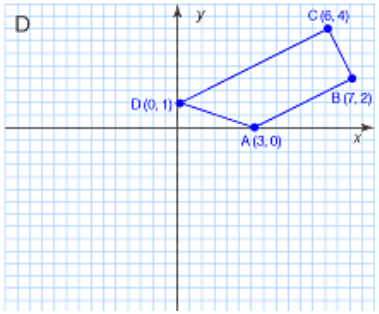
B



C



D



4) (*Answer feedback. Appears when button is pushed)*
Answer Feedback
The *y* values have remained the same, and the *x* values have become the opposites of those in the original figure.

The *y* values have remained the same, and the *x* values have become the opposites of those in the original figure.

5) (Answer feedback for incorrect answers)
[If A, C, or D is chosen:]

Sorry, you are not correct.
Remember that a reflection about the *y*-axis changes only the value of the *x*-coordinates. Please try again.

Remember that a reflection about the *y*-axis changes only the value of the *x*-coordinates. Please try again.

6) (incorrect feedback #2)
The correct answer is B. Notice that the *y* values have remained the same and the *x* values have become the opposites of those in the original figure.

The correct answer is B. Notice that the *y* values have remained the same, and the *x* values have become the opposites of those in the original figure.

Concept: Reflections m03p01

Practice Exercises

ID = m03p01

Larson_ca/chap2/sect5/prob2
Larson_ca/chap2/sect5/prob3

Tutorial: Transformations of Graphs and Functions m03v01

Video Clip Specification				
Filename: il0034m03v01.mov				
DVD/Book Title: Intermediate Algebra: Graphs and Functions				
Author: Larson				
Edition: 3				
Chapter & Section: Lesson 2.6: Transformation of Functions				
Duration: 4:47				
Clip Information:	**DVD Title**	**DVD Chapter**	**Time**	**Verbal Cue**
In	4	6	0:15:26	(beginning of animation) "We can examine the transformation idea just by using function notation…."
Out	4	6	0:20:13	… so here's our graph. (end of section)

Pedagogical Tags: For the video
Graphing Calculator: No
Applications: none
ISBN's: 0-618-39184-3, 0-618-15686-0, 0-618-10337-6, 0-618-38836-2, 0-618-38826-5, 0-618-38845-1

Tutorial: Transformations of Graphs and Functions mas01

MASTERY TESTS
ID= mas01

Reflections
il0032c01mas01.xml

Intermediate Algebra: Discovery and Visualization, 3/E [HMHI]
Hubbard/Robinson; 0-618-22381-9

1. 2.2.2.89
2. 9.8.1.310

Intermediate Algebra: Graphs & Functions [HMIG]

Larson; 0-618-21883-1

1. 2.6.2.67
2. 2.6.2.68
3. 2.6.2.69
4. 8.4.1.28
5. 8.4.1.29
6. 8.4.2.32
7. 8.7.1.55

Vertical Translations (or Shifts)
il0032c02mas01.xml

Algebra: Introductory and Intermediate, 3/E [HMII or 4886]
Aufmann/Barker/Lockwood; 0-618-29398-1

1. 11.2.1.11
2. 11.2.1.12

3 11.3.1.31

Horizontal Translations (or Shifts)
il0032c03mas01.xml

Intermediate Algebra with Applications [HMIP]
Aufmann; 0-618-30623-4

1. 8.6.1.51
2. 8.6.1.52
3. 9.1.1.1
4. 9.2.1.6
5. 9.2.1.7